

## CSIR-NET Full length TEST PAPER

# PHYSICS

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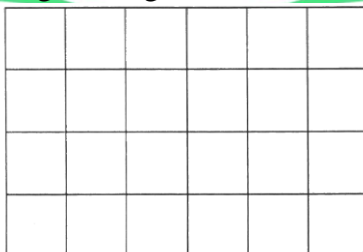
## ATS-4

(Full Length-C.M. & TS.)

### PART-A

**Directions:** - The Boss asked Janki to go and buy three different types of ball points pen. The first type costs Rs. 0.50 each, the second type Rs. 5.50 each and the third type Rs. 9.50 each. However the boss gave Janki only Rs. 100 with the strict instructions to buy exactly 100 pens, in any combination for that amount.

- Which Of The Following Is True About The Number Of Pens That Janki can buy
  - The number of pens of first type is the smallest
  - The number of pens of second type is the smallest
  - The number of pens of third type is smallest
  - The number of pens of the second type is largest
- The total cost of the first type of pens Janki should buy-
  - Is less than Rs. 40
  - Is more than Rs. 40 but less than Rs. 50
  - Is exactly Rs. 50
  - Is exactly Rs. 40
- If the square of a 2 digit number EB is MPB and given that M, E, P and B are all distinct numbers, how many possible values can P take?
  - 1
  - 2
  - 3
  - 4
- How many squares are in given figures :




- 50
  - 40
  - 60
  - 55
- A sphere of radius 4 cm. is curved from a homogenous sphere of radius 8 cm and mass 160 gram. The mass of the smaller sphere is :
    - 80
    - 60


**199-C,1<sup>st</sup> floor,opp.-bhandari hospital, Basant bahar colony, gopalpura mode, Jaipur-8769828844**

- (c) 40 (d) 20
6. The calendar for 1996 is the same for  
 (a) 2003 (b) 2012  
 (c) 2001 (d) 2024
7. If there are 12 person b/w A and B, 7 seven person b/w B and C, 4 person b/w C and D, and 5 person b/w D an E. How many minimum person are required for this arrangement :  
 (a) 14 (b) 16  
 (c) 33 (d) 25


8. Which of the position cannot be a standard Dice




(a)



(b)



(c)



(d)

9. One term in 1<sup>st</sup> series are wrong. Determine it and then give your answer according to 1<sup>st</sup> series :

I	1	3	7	11	21	43	85
II	2	(a)	(b)	(c)	(d)	(e)	(f)

- What will come in place of (f)-  
 (a) 149 (b) 124  
 (c) 75 (d) 168
10. Findt he odd Alphabet-  
 A E I O  
 (a) A (b) I  
 (c) E (d) O
11. How many 5 digit numbers can be formed by using the first 9 natural numbers, such that all the 5 digit of the numbers are formed in ascending order and at the hundredth place, the digit in '5'?  
 (a) 31 (b) 34  
 (c) 36 (d) 37
12. Jackie is twice as good a workman (i.e. twice as efficient) as Chan and takes 20 days less than Chan to complete a job. Find the time in which they can complete the job while working together :  
 (a)  $6\frac{2}{3}$  days (b)  $13\frac{1}{3}$  days  
 (c)  $15\frac{2}{3}$  days (d)  $16\frac{1}{3}$  days

13. Sajid's and Ankit's present ages are in the ratio 5 : 6. Three years ago their ages were in ratio 9 : 11. What will be the ratio of their ages after 6 years from now?  
 (a) 1 : 2 (b) 3 : 4  
 (c) 6 : 7 (d) 7 : 8
14. Which of the following is the largest?

- (a)  $\frac{1}{7^3}$  (b)  $\frac{1}{6^2}$   
(c)  $\frac{1}{8^4}$  (d)  $\frac{1}{5^6}$
15. If  $A = 123^{321} \times 371^{137} \times 326^{623} \times 347^{743} \times 112^{221}$ . Find the unit digit of A  
(a) 2 (b) 4  
(c) 8 (d) 6

16. What is the ratio of  $(a + b)$  to  $(a - b)$ ?  
(a)  $(a - b) : (a + b)$  (b)  $a^2 : b^2$   
(c)  $a : b$  (d)  $b : a$
17. What is the value of :  $\frac{2}{5}$  of  $\left(\frac{5}{3} \times \frac{6}{7}\right) \div \frac{4}{13} \times \frac{8}{13} = ?$   
(a)  $\frac{169}{56}$  (b)  $\frac{8}{7}$   
(c)  $\frac{28}{15}$  (d)  $\frac{22}{13}$
18. Two scooters are sold at Rs. 20,000 each. One of the scooters is sold at a profit of 10% while the other is sold at a loss of 10%. What is the overall profit or loss?  
(a) Loss of Rs. 404 (b) Profit of Rs. 202  
(c) Loss of Rs. 202 (d) No Profit, No Loss
19. There are 17 coins in a bag numbered from 1 to 17. Raju picks up a coin at random and puts it back in the bag. He continue this process until he gets a coin which shows an odd number. What is the probability that the coin shows a prime number when he stops?  
(a)  $\frac{1}{2}$  (b)  $\frac{1}{3}$   
(c)  $\frac{2}{3}$  (d)  $\frac{4}{5}$
20. "X", "Y" and "Z" are 3 runners who start running simultaneously around a square track ABCD of perimeter 70 meters from the point 'A', 'X' and 'Y' run anti-clockwise at speeds of 5 m/sec and 7 m/sec, respectively, whereas 'Z' runs clockwise at a speed of 9 m/sec. The ratio of the time taken by them to meet for the first time at the starting point to that of their first meeting anywhere on the track is :  
(a) 2 : 1 (b) 3 : 1  
(c) 3 : 2 (d) 1 : 9

### PART-B

21. For a system of two bosons, each of which can occupy any of the two energy levels 0 and  $\epsilon$ , the mean energy of the system at a temperature T with

$\beta = \frac{1}{k_B T}$  is given by

(a)  $\frac{\epsilon e^{-\beta\epsilon} + 2\epsilon e^{-2\beta\epsilon}}{1 + 2e^{-\beta\epsilon} + e^{-2\beta\epsilon}}$

(b)  $\frac{1}{2e^{-\beta\epsilon} + e^{-2\beta\epsilon}}$

(c)  $\frac{2\epsilon e^{-\beta\epsilon} + \epsilon e^{-2\beta\epsilon}}{2 + e^{-\beta\epsilon} + e^{-2\beta\epsilon}}$

(d)  $\frac{\epsilon e^{-\beta\epsilon} + 2\epsilon e^{-2\beta\epsilon}}{2 + e^{-\beta\epsilon} + e^{-2\beta\epsilon}}$

22. A particle motion on a space – curve is governed by  $x = 2 \sin t$ ,  $y = 3 \cos t$  and  $z = \sqrt{5} \sin t$ . What is the magnitude of the velocity of the particle at any time  $t$ ?

(a)  $3\sqrt{2} \sin t$

(b) 3

(c)  $3\sqrt{2} \cos t$

(d)  $3\sqrt{2}$

23. A rod is moving with velocity  $0.8 c$  in a direction inclined at  $60^\circ$  to its own length. The rod will contract by

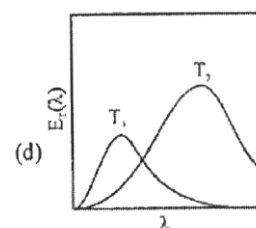
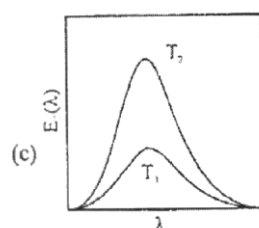
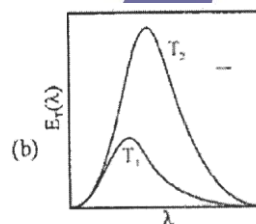
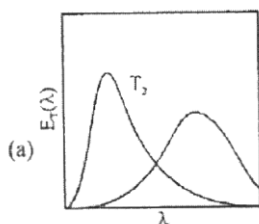
(a) 4.15%

(b) 8.3%

(c) 12.45%

(d) 16.6%

24. Which of the graphs below gives the correct qualitative behavior of the energy density  $E_T(\lambda)$  of blackbody radiation of wavelength  $\lambda$  at two temperatures  $T_1$  and  $T_2$  ( $T_1 < T_2$ )?



25. The root-mean square velocity  $\bar{v}$  of helium atom at  $T$  degree K satisfies the relation

(a)  $\bar{v} \propto \sqrt{T}$

(b)  $\bar{v} \propto \frac{1}{\sqrt{T}}$

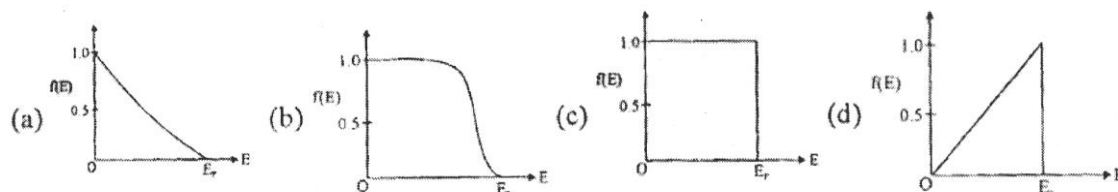
(c)  $\bar{v} \propto T$

(d)  $\bar{v} \propto \frac{1}{T}$

26. Which of the following statement best explains why the specific heat of electrons in metals is much smaller than that expected in a non-interacting (free) electron gas model?

(a) The mass of electron is much smaller than that of the ions in the crystal.

- (b) The Pauli exclusion principle restricts the number of electrons which can absorb thermal energy  
 (c) Electron spin can take only two different values.  
 (d) Electrons in a metal cannot be modeled as non-interacting.
27. A gas of photons is enclosed in a container of fixed volume at an absolute temperature  $T$ . Noting that the photon is massless particle (i.e. its energy and momentum are related by  $E = pc$ ), the number of photons in the container will vary as
- (a)  $T$  (b)  $T^2$   
 (c)  $T^3$  (d)  $T^4$
28. Consider the melting transition of ice into water at constant pressure. Which of the following thermodynamic quantities does not exhibit a discontinuous change across the phase transition?  
 (a) Internal energy (b) Helmholtz free energy  
 (c) Gibbs free energy (d) Entropy
29. A vessel has two compartments of volume  $V_1$  and  $V_2$ , containing an ideal gas at pressures  $p_1$  and  $p_2$ , and temperatures  $T_1$  and  $T_2$  respectively. If the wall separating the compartments is removed, the resulting equilibrium temperature will be
- (a)  $\frac{p_1 T_1 + p_2 T_2}{p_1 + p_2}$  (b)  $\frac{V_1 T_1 + V_2 T_2}{V_1 + V_2}$   
 (c)  $\frac{p_1 V_1 + p_2 V_2}{(p_1 V_1 / T_1) + (p_2 V_2 / T_2)}$  (d)  $(T_1 T_2)^{1/2}$
30. If  $P_1$ ,  $P_2$  and  $P_3$  represent the pressures of ideal B.E.; M.B. and F.D. gases at low temperature, respectively then  
 (a)  $P_1 < P_2$ ;  $P_1 < P_3$  (b)  $P_1 < P_3$ ;  $P_1 > P_2$   
 (c)  $P_1 > P_2$ ;  $P_1 > P_3$  (d)  $P_1 = P_2 = P_3$
31. Two localized non-interacting spin  $1/2$  magnetic ions of magnetic moment  $\mu$  are placed in an external magnetic field  $H$ , at temperature  $T$ . If  $k_B T \gg \mu H$ , then the entropy of the system is, to a good approximation.  
 (a)  $S = k_B \ln 2$  (b)  $S = 2k_B \ln 2$   
 (c)  $S = 3k_B \ln 2$  (d)  $S = 4k_B \ln 2$
32. According to Fermi-Dirac statistics, the probability of occupation of a quantum state of energy  $E$  is  $f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$  where  $E_F$  is the Fermi energy. Which of the following graphs is true at  $T = 0K$ .



33. The number of ways in which 5 identical bosons can be distributed in 4 states is :

(a)  $\frac{8!}{5!3!}$  (b)  $\frac{9!}{5!4!}$   
 (c)  $\frac{9!}{4!4!}$  (d)  $\frac{8!}{4!4!}$

34. A system consists of two Bosons each of which can occupy two energy levels 0 and  $E_0$ . The partition function for the system is :

(a)  $1 + e^{-\beta E_0}$  (b)  $1 + e^{-2\beta E_0}$   
 (c)  $1 + e^{-\beta E_0} + e^{-2\beta E_0}$  (d)  $e^{-\beta E_0} + e^{-2\beta E_0}$ , where  $\beta = \frac{1}{kT}$

35. A particle of spin  $\frac{1}{2}$  and magnetic moment  $\mu$  is in a magnetic field B. Its partition functions is :

(a)  $2 \tanh \frac{\mu B}{kT}$  (b)  $2 \cosh \frac{\mu B}{kT}$   
 (c)  $2 \sinh \frac{\mu B}{kT}$  (d)  $e^{\frac{\mu B}{kT}}$

36. A gas of N non-interacting particles is in thermal equilibrium at temperature T. Each particle can be in any of the possible non-degenerate states of energy 0,  $2\epsilon$  and  $4\epsilon$ . The average energy per particle of the gas, when  $\beta\epsilon \ll 1$ , is :

(a)  $2\epsilon$  (b)  $3\epsilon$   
 (c)  $2\epsilon/3$  (d)  $\epsilon$

37. Consider the Fermi-Dirac distribution function  $f(E)$  at room temperature (300K) where E refers to energy. If  $E_F$  is the Fermi energy, which of the following is true?

(a)  $f(E)$  is a step function (b)  $f(E_F)$  has a value of 1/2  
 (c) States with  $E < E_F$  are filled completely  
 (d)  $f(E)$  is large and tends to infinity as E decreases much below  $E_F$

38. Thermodynamic variables of a system can be volume V, pressure P, temperature T, number of particles N, internal energy E and chemical potential  $\mu$ , etc. For a system to be specified by microcanonical (MC), Canonical (CE) and Grand Canonical (GC) ensembles, the parameters required for the respective ensembles are

- (a)  $MC : (N, V, T)$ ;  $CE : (E, V, N)$ ;  $GC : (V, T, \mu)$   
 (b)  $MC : (E, V, N)$ ;  $CE : (N, V, T)$ ;  $GC : (V, T, \mu)$   
 (c)  $MC : (V, T, \mu)$ ;  $CE : (N, V, T)$ ;  $GC : (E, V, N)$   
 (d)  $MC : (E, V, N)$ ;  $CE : (V, T, \mu)$ ;  $GC : (N, V, T)$

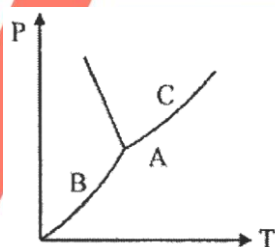
39. A thermally-insulated container of volume  $V_0$  is divided into two equal halves by a non-permeable partition. A real gas with equation of state.

$$b^3 \left( p + \frac{a^2}{V^3} \right) = nRT$$

where 'a' and 'b' are constants, is confined to one of these halves at a temperature  $T_0$ . The partition is now removed suddenly and the gas is allowed to expand to fill the entire container. The final temperature of the gas, in terms of its specific heat  $C_V$ , will be

- (a)  $T_0 - \frac{3a^2}{2C_V V_0^2}$  (b)  $T_0 - \frac{2a^2}{3C_V V_0^2}$   
 (c)  $T_0 + \frac{3a^2}{2C_V V_0^2}$  (d)  $T_0 + \frac{2a^2}{3C_V V_0^2}$

40. The pressure versus temperature diagram for a pure substance is shown below :



The region represented by A, B & C respectively are

- (a) Solid, liquid and vapour (b) Vapour, solid and liquid  
 (c) Solid, vapour and liquid (d) Vapour liquid and solid.
41. Consider a perfect gas in a vessel rushing into an evacuated vessel and let the whole system be isolated. Let  $V_1$  and  $V_2$  denote the initial and final volumes of the gas. The increase in entropy of the system is :

- (a) Zero (b)  $R \ln \frac{V_2}{V_1}$

- (c)  $\frac{1}{T} \int_{V_1}^{V_2} du + R \ln \frac{V_2}{V_1}$  (d)  $\frac{1}{T} \int_{V_1}^{V_2} du$

42. A simple calculation for the entropy  $S$  of an ideal gas gives  $S = \frac{3}{2} Nk_B \ln V + S_0$  where  $N$  is the number of molecules in the gas,  $V$  is its

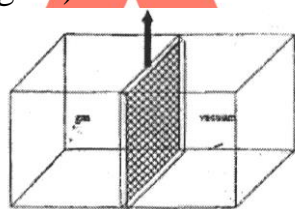
volume, and  $S_0$  an arbitrary constant. This expression is erroneous because :

- (a) It does not contain temperature T
- (b) It does not contain pressure P
- (c) It is not extensive
- (d) It does not take into account the interaction between the molecules of the gas.

43. Two insulated containers connected with a stop cock are initially evaluate. An ideal gas is filled in one of them and then allowed to expand into the other container by opening the cock. In this process the work done (W), the heat evolved (Q) and the change in internal energy ( $\Delta U$ ) are given by

- (a)  $W = 0, Q = 0$  and  $\Delta U = 0$
- (b)  $W = +ve, Q = 0$  and  $\Delta U = -ve$
- (c)  $W = -ve, Q = 0$  and  $\Delta U = -ve$
- (d)  $W = -ve, Q = 0$  and  $\Delta U = +ve$

44. Consider a sealed but thermally conducting container of total volume V, which is in equilibrium with athermal bath at temperature T. The container is divided into two equal chambers by a thin but impermeable partition. One of these chambers contains an ideal gas, while the other half is a vacuum (see figure).



If the partition is removed and the ideal gas is allowed to expand and fill the entire container, then the entropy per molecule of the system will increase by an amount

- (a)  $2k_B$
- (b)  $k_B \ln(1/2)$
- (c)  $k_B \ln 2$
- (d)  $(k_B \ln 2) / 2$

45. A steam turbine is operated with an intake temperature  $407^\circ\text{C}$  and exhaust temperature  $117^\circ\text{C}$ . Heat input to the turbine is Q. then maximum work turbine can do is :

- (a)  $0.5Q$
- (b)  $0.37Q$
- (c)  $Q$
- (d)  $0.25 Q$

### PART-C

46. Consider a system, at thermal equilibrium at temperature T, with energy levels  $E_j = J\varepsilon$ , ( $J = 0, 1, 2, 3, \dots, \infty$ ) and N distinguishable particles. If the

mean energy per particle is  $a\varepsilon$ , then the inverse temperature  $\left( \beta = \frac{1}{k_B T} \right)$

of the system is given by



(a)  $\frac{1}{\varepsilon} \ln \left( 1 - \frac{1}{a} \right)$  (b)  $\frac{1}{\varepsilon} \ln \left( 1 + \frac{1}{a} \right)$

(c)  $\frac{1}{\varepsilon} \ln (1 - a)$  (d)  $\frac{1}{\varepsilon} \ln (1 + a)$

47. The Hamiltonian for three Using spins  $S_1, S_2$  and  $S_3$ , taking values  $\pm 1$  is  $H = -J(S_1S_2 + S_2S_3 + S_3S_1)$

If the system is in thermal equilibrium at temperature  $T$ , the average energy of the system when  $J / k_B T = 1/4$ , is

(a)  $3 \left( \frac{e+1}{e-3} \right)$  (b)  $-3 \left( \frac{e+1}{e-3} \right)$

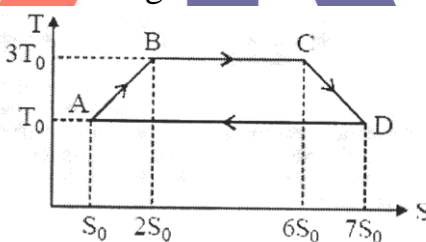
(c)  $3 \left( \frac{e-1}{e+3} \right)$  (d)  $-3 \left( \frac{e-1}{e+3} \right)$

48. Consider a system which is in thermal equilibrium at temperature  $T$ . If the system has one atom which may or may not occupy any of the one energy level out of the given three levels having energy  $0, \varepsilon$  and  $2\varepsilon$ . The average energy of the system if  $\mu$  is the chemical potential of the system is :

(a)  $\frac{\varepsilon e^{-\beta\mu} (e^{-\beta\varepsilon} + e^{-2\beta\varepsilon})}{1 + e^{-\beta\mu} (1 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon})}$  (b)  $\frac{\varepsilon e^{\beta\mu} (e^{-\beta\varepsilon} + e^{-2\beta\varepsilon})}{1 + e^{\beta\mu} (1 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon})}$

(c)  $\frac{\varepsilon e^{-\beta\mu} (e^{-\beta\varepsilon} + 2e^{-2\beta\varepsilon})}{1 + e^{-\beta\mu} (1 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon})}$  (d)  $\frac{\varepsilon e^{\beta\mu} (e^{-\beta\varepsilon} + 2e^{-2\beta\varepsilon})}{1 + e^{\beta\mu} (1 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon})}$

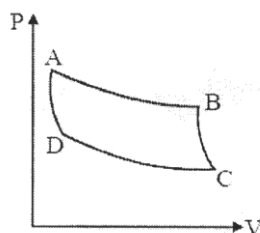
49. The T-S diagram of an ideal gas is shown below :



The efficiency of the device run on the given cyclic process is :

- (a)  $5/7$  (b)  $3/7$   
(c)  $5/6$  (d)  $5/8$

50. An engine working on a Carnot cycle is shown below :



If the working substance of the engine is a diatomic ideal gas and the volume of the gas at point C is 32 times the volume of the gas at point B, the efficiency of the engine is :

- (a) 0.50 (b) 0.25  
(c) 0.85 (d) 0.75

51. If one mole of an ideal gas expands isothermally at temperature  $T$  from initial volume  $V_0$  to final volume  $2V_0$ , the change in Helmholtz free energy is :

- (a)  $-RT \ln 2$  (b)  $RT \ln 2$   
(c)  $RT$  (d) 0

52. There are 5 bosons to be distributed in 4 energy levels and 4 fermions to be distributed in 5 energy levels. The ratio of the number of ways of distributing bosons to that of fermions is :

- (a) 7 : 12 (b) 14 : 1  
(c) 56 : 5 (d) 7 : 15

53. When a collection of two-level system is in equilibrium at temperature  $T_0$ . The ratio of the population in the lower and upper level is 3 : 1, when the temperature is changed to  $T$ . The ratio is 9 : 1, then

- (a)  $T = 2T_0$  (b)  $T_0 = 2T$   
(c)  $T_0 = 3T$  (d)  $T_0 = 4T$

54. The average local internal magnetic field acting on a spin is  $H_{int} = \alpha M$ , where  $M$  is the magnetization and  $\alpha$  is a positive constant. At a temperature  $T$  sufficiently close to (and above) the critical temperature  $T_c$ , the magnetic susceptibility at zero external field is proportional to ( $k_B$  is the Boltzmann constant)

- (a)  $k_B T - \alpha$  (b)  $(k_B T + \alpha)^{-1}$   
(c)  $(k_B T - \alpha)^{-1}$  (d)  $\tan h (k_B T + \alpha)$

55. The dynamics of a particle of mass  $m$  is described in terms of three generalized coordinates  $\xi$ ,  $\eta$  and  $\phi$ . If the lagrangian of the system is

$$L = \frac{1}{8} m \left[ (\xi + \eta) \left( \frac{\dot{\xi}^2}{\xi^2} + \frac{\dot{\eta}^2}{\eta^2} \right) + 4\eta\dot{\phi}^2 \right] + \frac{1}{2} K (\xi + \eta)^2 \quad \text{where } K \text{ is a}$$

constant, then a conserved quantity in the system will be

- (a)  $(m + k)(\dot{\xi} + \dot{\eta})$  (b)  $m\xi\eta\dot{\phi}$   
(c)  $m \left( \frac{\dot{\xi}^2}{\eta^2} + \frac{\dot{\eta}^2}{\xi^2} \right)$  (d)  $m(\xi + \eta) \left( \frac{\dot{\xi}}{\xi^2} + \frac{\dot{\eta}}{\eta^2} \right)$

56. A particle of mass  $m$  moves in the  $xy$  plane in the presence of a potential  $V(x,y)$  so that its lagrangian is given by

$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}(x^2 + y^2) + 21n|x - y|$  which of following statements corresponds to the equilibrium of the system.

- (a) There is no stable equilibrium at any finite values of  $(x, y)$   
 (b) There is only one stable equilibrium at the point  $(x, y) = (0, 0)$   
 (c) There are two stable equilibrium at the points  $(x, y) = (1, 1)$  and  $(-1, -1)$   
 (d) There are two stable equilibrium at the points  $(x, y) = (1, -1)$  and  $(-1, 1)$

57. An ensemble of quantum harmonic oscillator is kept at a finite temperature  $T = \frac{1}{K_B\beta}$  energy level of a single oscillator is  $(n + \frac{1}{2})\hbar\omega$ , specific heat varies with temperature (T) at  $K_B T \gg \hbar\omega$  is

- (a) T  
 (b)  $e^{-\hbar\omega/K_B T}$   
 (c)  $T^3$   
 (d) independent of T.

58. Consider the differential operator  $\frac{dy}{dx} = e^{-4t}$  with the initial condition  $y(0) = 2$ . Then the Laplace transform  $Y(S)$  of the solution  $y(t)$  is :

- (a)  $\frac{2S+9}{S(S+4)}$   
 (b)  $\frac{(2S+3)}{S(S+2)}$   
 (c)  $\frac{2S}{S(S+1)}$   
 (d)  $\frac{2S+9}{S(S+4)}$

59. The time period of a simple pendulum under the influence of the acceleration due to gravity  $g$  is  $T$ . The bob is subjected to an additional acceleration of magnitude  $\sqrt{3}g$  in the horizontal direction. Assuming small oscillations, the mean position and time period of oscillation, respectively, of bob will be

- (a)  $0^\circ$  to the vertical and  $\sqrt{3}T$   
 (b)  $30^\circ$  to the vertical and  $T/2$   
 (c)  $60^\circ$  to the vertical and  $T/\sqrt{2}$   
 (d)  $0^\circ$  to the vertical and  $T/\sqrt{3}$

60. A system can have three energy levels :  $E = 0, \pm \epsilon$ , The level  $E = 0$  is doubly degenerate, while the others are non-degenerate. The average energy at inverse temperature  $\beta$  is

- (a)  $-\epsilon \tanh(\beta\epsilon)$   
 (b)  $\frac{\epsilon(e^{\beta\epsilon} - e^{-\beta\epsilon})}{(1 + e^{\beta\epsilon} + e^{-\beta\epsilon})}$   
 (c) Zero  
 (d)  $-\epsilon \tanh\left(\frac{\beta\epsilon}{2}\right)$

61. Let  $(q, p)$  denote the position and momentum pair in phase space of a classical system consider the transformation is given  $(q, p) \rightarrow (Q, P)$

$$Q = \alpha q^m \quad p = \beta q^n$$

$\alpha, \beta, m, n$  are constant. Value of  $m$  and  $n$  such that  $(q, p) \rightarrow (Q, P)$  is canonical transformation

- (a)  $m = 1 - \frac{1}{\alpha\beta}, n = -\frac{1}{\alpha\beta}$                       (b)  $m = 1 + \frac{1}{\alpha\beta}, n = \frac{1}{\alpha\beta}$   
 (c)  $m = 1 + \frac{1}{\alpha\beta}, n = -\frac{1}{\alpha\beta}$                       (d)  $m = \frac{1}{\alpha\beta}, n = -\frac{1}{\alpha\beta}$

62. The Lagrangian of a system described by a single generalized coordinate  $q$  is  $L = \frac{1}{2} \dot{q} \sin^2 q$

Its Hamiltonian is

- (a) not defined                      (b) Zero  
 (c)  $-\dot{q} \sin^2 q$                       (d)  $\dot{q} \left( p - \frac{1}{2} \sin^2 q \right)$

63. A classical particle with total energy  $E$  moves under the influence of a potential  $V(x, y) = 3x^3 + 2x^2y + 2xy^2 + y^3$ . The average potential energy, calculated over a long time is equal to,

- (a)  $\frac{2E}{3}$                       (b)  $\frac{E}{3}$   
 (c)  $\frac{E}{5}$                       (d)  $\frac{2E}{5}$

64. The Lagrangian of a particle is given by  $L = \dot{q}^2 - q\dot{q}$ . Which of the following statements is true?

- (a) This is a free particle  
 (b) The particle is experiencing velocity dependent damping  
 (c) The particle is executing simple harmonic motion  
 (d) The particle is under constant acceleration.

65. A dynamical system with two generalized coordinates  $q_1$  and  $q_2$  has Lagrangian  $L = \dot{q}_1^2 + \dot{q}_2^2$ . If  $p_1$  and  $p_2$  are the corresponding generalized momenta, the Hamiltonian is given by

- (a)  $(p_1^2 + p_2^2) / 4$                       (b)  $(\dot{q}_1^2 + \dot{q}_2^2) / 4$   
 (c)  $(p_1^2 + p_2^2) / 2$                       (d)  $(p_1\dot{q}_1 + p_2\dot{q}_2) / 4$

66. In a certain inertial frame two light pulses are emitted, a distance 5 km apart and separated by  $5\mu\text{s}$ . An observer who is traveling, parallel to the line joining the points where the pulses are emitted, at a velocity  $V$  with

respect to this frame notes that the pulses are simultaneous. Therefore  $V$  is

- (a)  $0.7 c$  (b)  $0.8 c$   
(c)  $0.3 c$  (d)  $0.9 c$

67. If the Poisson bracket  $\{x, p\} = -1$ , then the Poisson bracket  $\{x^2 + p, p\}$  is?

- (a)  $-2x$  (b)  $2x$   
(c)  $1$  (d)  $-1$

68. If the coordinate  $q$  and the momentum  $p$  form a canonical pair  $(q, p)$ , which one of the sets given below also forms a canonical?

- (a)  $(q - p)$  (b)  $(q^2, p^2)$   
(c)  $(p, -q)$  (d)  $(q^2, -p^2)$

69. A two dimensional box in a uniform magnetic field  $B$  contains  $\frac{N}{2}$  localized spin  $-\frac{1}{2}$  particles with magnetic moment  $\mu$ , and  $\frac{N}{2}$  free spinless particles which do not interact with each other. The average energy of the system at a temperature  $T$  is :

- (a)  $3NkT - \frac{1}{2}N\mu B \sinh\left(\frac{\mu B}{k_B T}\right)$  (b)  $NkT - \frac{1}{2}N\mu B \tanh\left(\frac{\mu B}{k_B T}\right)$   
(c)  $\frac{1}{2}NkT - \frac{1}{2}N\mu B \tanh\left(\frac{\mu B}{k_B T}\right)$  (d)  $\frac{3}{2}NkT + \frac{1}{2}N\mu B \cosh\left(\frac{\mu B}{k_B T}\right)$

70. An ideal gas has a specific heat ratio  $\frac{C_P}{C_V} = 2$ . Starting at a temperature  $T_1$  the gas under goes an isothermal compression to increase its density by a factor of two. After this an adiabatic compression increases its pressure by a factor of two. The temperature of the gas at the end of the second process would be:

- (a)  $\frac{T_1}{2}$  (b)  $\sqrt{2}T_1$   
(c)  $2T_1$  (d)  $\frac{T_1}{\sqrt{2}}$

71. A particle in thermal equilibrium has only 3 possible states with energies  $-\epsilon, 0, \epsilon$ . If the system is maintained at a temperature  $T \gg \frac{\epsilon}{k_B}$ , then the

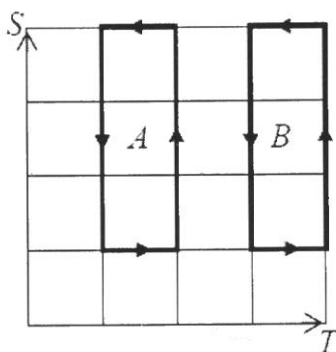
average energy of the particle can be approximated to,

- (a)  $\frac{2\epsilon^2}{3k_B T}$  (b)  $\frac{-2\epsilon^2}{3k_B T}$

(c)  $\frac{-\epsilon^2}{k_B T}$  (d) 0

72. The blackbody at a temperature of 6000 K emits a radiation whose intensity spectrum peaks at 600 nm. If the temperature is reduced to 300K, the spectrum will peak at,  
 (a) 120μm (b) 12μm  
 (c) 12mm (d) 120mm

73. The entropy-temperature diagram of two Carnot engines, A and B, are shown in the figure 4. The efficiencies of the engines are  $\eta_A$  and  $\eta_B$  respectively. Which one of the following equalities is correct?



- (a)  $\eta_A = \frac{\eta_B}{2}$  (b)  $\eta_A = \eta_B$   
 (c)  $\eta_A = 3\eta_B$  (d)  $\eta_A = 2\eta_B$
74. A monoatomic gas consists of atoms with two internal energy levels, ground state  $E_0 = 0$  and an excited state  $E_1 = E$ . The specific heat of the gas is given by

(a)  $\frac{3}{2}k$  (b)  $\frac{E^2 e^{E/kT}}{kT^2 (1 + e^{E/kT})^2}$   
 (c)  $\frac{3}{2}k + \frac{E^2 e^{E/kT}}{kT^2 (1 + e^{E/kT})^2}$  (d)  $\frac{3}{2}k - \frac{E^2 e^{E/kT}}{kT^2 (1 + e^{E/kT})^2}$

75. Consider a system of two particles A and B. Each particle can occupy one of three possible quantum states  $|1\rangle, |2\rangle$  and  $|3\rangle$ . The ratio of the probability that the two particles are in the same state to the probability that the two particles are in different states is calculated for bosons and classical (Maxwell-Boltzmann) particles. They are respectively.

(a) 1, 0 (b)  $\frac{1}{2}, 1$   
 (c)  $1, \frac{1}{2}$  (d)  $0, \frac{1}{2}$